Job Assignment with Multivariate Skills and the Peter Principle *

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Abstract

This paper analyzes the job assignment problem faced by a firm when workers' skills are distributed along several dimensions and jobs require different skills to varying extent. I derive optimal assignment rules with and without slot constraints, and show that under certain circumstances workers may get promoted although they are expected to be less productive in their new job than in their old job. This can be interpreted as a version of the Peter Principle which states that workers get promoted up to their level of incompetence.

Keywords: job assignment, worker selection, internal hiring, Peter Principle, slot constraints, multi-dimensional skills.

1 Introduction

How should a firm assign workers to jobs? This basic question may be rather difficult to answer in a context where workers' performance depends on a

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large number of different skills that are hard to measure individually. In most jobs, the performance of a worker depends on many things, such as his level of expertise, his capacity to concentrate and organize, as well as his analytical and communication skills. As workers switch jobs or move up in the hierarchy, the relative importance of each of these skills changes. Yet, employers often can only observe an aggregate performance measure on which to base their decisions when allocating tasks or deciding on promotions. So, given such a setup, which workers should get reassigned or promoted? When can performance in one job act as an indicator for performance in another job?

To answer these questions, I concentrate on learning by the employer about the skill portfolio of his employees.[1] I also abstract from any considerations concerning wage costs or workers' incentives to exert effort. Instead, the paper focuses on the employer's task assignment problem when workers vary in their skill levels and different tasks require different combinations of skills. This allows me to discuss optimal assignment rules for workers and how different jobs should be ordered within a job rotation scheme or a hierarchy. Furthermore, the model offers a new explanation for the Peter Principle in showing that internal promotions may be efficient even if the expected output after promotion is lower than the current output, due to information problems.

Workers in the model are characterized by a skill combination (x, z) where x and z are two different skills that are independently distributed across the worker population.[2] However, only a worker's overall performance in a given task is observable, not his exact skill profile. The employer therefore faces the problem of how to allocate workers to tasks that put different weights on both skills.[3]

The first part of the paper analyzes the task assignment of current and new workers if the employer faces no constraints concerning the number of workers he can assign to each task. I derive assignment rules and show that there may be a tradeoff between maximizing short-term and long-term output when new workers are hired for two periods: in the short run, output is maximized by

[1]Obviously, human capital formation and on the job training may also have an important impact on sequential job assignments and promotions, as for instance discussed in the seminal paper by Gibbons and Waldman (1999).

[2]For instance, x could be analytical skills and z communication skills.

[3]Suppose for instance, an accounting firm needs people to crunch the numbers and to write reports. Both tasks require some analytical and communication skills, but to varying extent. Then the firm faces two questions: (1) where to put new workers, and (2) after having observed worker's performance in one of the two tasks, whether to reallocate them to the other.

assigning new workers to the task where the expected output of an unscreened worker is maximal. However, if this task plays a much more important role in the overall production of the firm, the employer may prefer first to hire workers for the other task, which is thus used as a screening stage for maximizing output in the long run. That is, the employer will prefer to forego some first-period output in order to make more informed choices in the long run.

The second part of the paper abandons the assumption of unconstrained assignment possibilities. At least in the short run, firms often need a fixed number of workers in each task, i.e., there is a given number of jobs in each activity. Given such slot constraints, the second part of the paper determines under what circumstances workers are reallocated between jobs. It thus describes situations in which firms prefer internal over external hiring of workers.

Furthermore, the analysis provides theoretical evidence for a version of the Peter Principle, after Peter and Hull (1969), which states that workers are promoted up to their level of incompetence. In the present model, something similar happens as workers may be reallocated although, in expected terms, they will be less productive at their new job. However, this reallocation is efficient, unlike suggested by the original Peter Principle, which is generally associated with inefficient reallocations. The intuition behind this result is as follows: Having observed a worker, albeit in a different job, gives the employer more information about his skill level than about potential outside candidates. The firm therefore may be better off reallocating a mediocre worker on whom it has at least some information, rather than hiring an unknown worker. In contrast to the original Peter Principle, this result is not limited to promotions, but applies more generally, both to vertical and lateral movements in a hierarchy.

The paper is structured as follows: the following section summarizes the related literature. Section 3 describes the optimal task assignment of workers when there are two tasks that require two skills, and there are no slot constraints. This latter assumption is relaxed in Section 4, which analyzes under which circumstances workers are reallocated between jobs if there is a fixed number of workers needed in each job. Additionally, Section 5 further generalizes the model, and Section 6 discusses assignment patterns such as job rotation. Section 7 concludes.

2 Related Literature

While there is a large literature on job assignment, as nicely summarized in Sattinger (1993) and Valsecchi (2000), the model discussed in this paper is closest to the papers by Waldman (1984) and Bernhardt (1995), in that it abstracts from strategic considerations of the employees. Both authors assume that the ability of an employee is only observed by his current employer, but not by potential competitors. As Waldman (1984) has shown, competition for high ability workers then may result in an inefficient assignment of workers to jobs since firms will be reluctant to reveal their private information about the worker's ability by promoting them. As discussed in Bernhardt (1995), this may help to explain a certain number of observations typically made in the labour market. Overall, firms' reluctance to promote will increase the higher the discrepancy between their private and the publicly available information on a worker, as reflected by his education and past promotion path. The resulting inefficiencies in job assignment are only mitigated if human capital is more firm specific.

Similar to Waldman (1984) and Bernhardt (1995), most papers on job assignment assume that workers' abilities vary along a single dimension ("general ability") but do not take into account that workers may possess many skills that matter for their performance. Others note that workers may have a comparative advantage in one task, but then often reduce the analysis to a problem with only two types. The present paper tries to take a more differentiated approach by introducing several a priori unobservable skills and thus putting the focus on the employer's learning through job assignment.

By considering several skills, the paper is also close to the concept of task-specific skills introduced by Gibbons and Waldman (2004, 2006). Furthermore, Lazear (2009) proposes a model with two skills that are used with varying intensity in different firms, thus reconsidering the meaning of firm-specific skills. Building on these papers, Gathmann and Schonberg (2010) analyze empirically to what extent skills are transferable across jobs, the underlying idea being that different occupations combine tasks and thus task-specific skills in different ways.

Like Gathmann and Schonberg (2010), the present paper uses the idea that workers possess many skills and that their skill profile matters for their performance in different tasks. It then derives conditions under which employers prefer to recruit workers internally.

There is a lot of empirical evidence that firms indeed tend to hire internally, as for instance shown in DeVaro and Morita (2009), in Agrawal, Knoeber, and

Tsoulouhas (2006), and in Lauterbach, Vu, and Weisberg (1999). The theoretical literature on this topic has proposed several explanations for this phenomenon. For instance, Chan (1996) stresses the incentives created through promotions as a reason for limiting external hiring which would decrease the chances of promotion of current employees.[4] Another explanation is given by Greenwald (1986), who proposes a model where employers prefer internal hiring because they know the ability of workers they have employed for one period. Finally, Demougin and Siow (1994) show that, with positive hiring costs, firms may prefer to train and screen workers themselves. By contrast, no such costs are needed in the present model to make internal recruiting attractive: it is enough that the employer has slightly better information about current employees compared to new ones.[5] Employer learning is also considered in Meyer (1991) who proposes to bias contests between employees in order to learn more about their respective abilities. While the author mentions the possibility of producing a bias through task differentiation, this possibility is not further discussed in the paper. But the idea is picked up by Carrillo (2003) who analyzes how job assignment can be used as a screening device. He shows that due to career concerns, contests should be biased in favor of outsiders and junior agents.[6]

Finally, this paper offers a new explanation for the Peter Principle. The existing literature explains the Peter Principle as a by-product of various incentive problems (Fairburn and Malcomson, 2001; Koch and Nafziger, 2012) or as a statistical artefact (Lazear, 2004), whereas this paper shows that the Peter Principle may even hold when such considerations play no role at all.

In Fairburn and Malcomson (2001), rewarding employees for performance is delegated to managers who are susceptible to yield to influence activities by employees. The use of promotions rather than bonus payments helps to mitigate this problem. However, if employees are risk-averse, promotion decisions may be distorted and result in inefficient assignments. In particular, the performance threshold for promotion may be lowered and too many employees will be promoted which is interpreted as evidence for the Peter

[4] Waldman (2003), in a similar setup, points out that the firm may face a time inconsistency problem: ex ante, the incentive aspect is more important, but at the date of promotion, i.e., when effort has already been spent, the assignment aspect becomes more important. That may cause a time inconsistency problem which can be resolved by establishing an internal labor market.

[5] Still, the model in Demougin and Siow (1994) is in some respects similar to the analysis presented here, for instance in its consideration of slot constraints.

[6] While the paper by Carrillo (2003) shows some similarity to the present paper, it is quite different in focus and makes a number of differing assumptions, such as considering the delegation of task allocation and allowing for effort by the agents.

Principle. Koch and Nafziger (2012) have a similar result. In their model, the probability that low effort leads to high output decreases as employees move up in the hierarchy. Higher level jobs are more informative about workers' effort, thus reducing the cost of incentives. This may outweigh the cost of a lower success probability that a wrong assignment of the agent has. As a result, the principal lowers the promotion threshold. Lazear (2004), on the other hand, explains the Peter Principle as the outcome of a statistical process that displays regression to the mean: A worker benefiting from a high temporary productivity shock will have higher output and be more likely to get promoted. However, over time, his productivity will return to its average value, thus giving the impression that the worker's productivity fell after promotion.[7]

By contrast, this paper abstracts from incentives and concentrates on the assignment problem. It shows that internal promotion may be efficient even if the expected output after promotion falls compared to the current output of the worker. Rather than being a statistical matter as in Lazear (2004) or a side effect of incentive problems as in Fairburn and Malcomson (2001) and Koch and Nafziger (2012), the Peter Principle hence arises as a consequence of incomplete information about job candidates: Instead of hiring an outsider, the employer may prefer to promote a worker on whom he has at least some information, even if the output of this worker in his current job is likely to be higher than after promotion.

3 Optimal Task Assignment

Suppose there are two kinds of activities or tasks in a firm. A worker engaged in either of these two activities produces an output y_j, $j = 1, 2$, according to the following production functions

$$
\begin{aligned}
y_1 &= \alpha \cdot x + (1-\alpha) \cdot z \\
y_2 &= \beta \cdot x + (1-\beta) \cdot z ,
\end{aligned}
$$

where x and z are two different skills needed in both activities, and α and β are the weights assigned to skill x, with $\alpha, \beta \in [0, 1]$.

Skills x and z can, for instance, be thought of as technical and analytical skills respectively. Let us assume that skill x is more important in activity 1

[7] See Barmby, Eberth, and Ma (2012) for an empirical test of the paper by Lazear (2004).

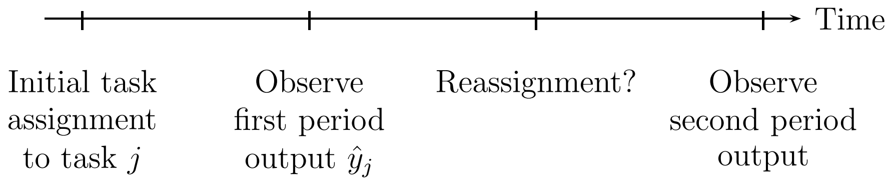

Figure 1: Time line.

than in activity 2, whereas it is the other way round for skill z. That is, the following assumption holds:

Assumption 1 $\alpha > \beta$.

The cross-section distribution of the two skills is assumed to be normal with $x \sim N(\mu_x, \sigma_x)$ and $z \sim N(\mu_z, \sigma_z)$, and zero correlation between the two skills.[8] These statistical properties are common knowledge. Furthermore, the employer can observe the output produced by a worker, but not the worker's specific skill level (x, z). The employer can hire workers for 2 periods of time at most and may reassign them after the first period. That is, upon hiring a worker he faces a sequence of decisions as shown in Figure 1.

For the rest of this section, it is assumed that the employer may assign any number of workers to any of the two tasks. That is, the employer faces no slot constraints in his allocation problem but can concentrate on the question how a given worker is optimally allocated to one of the two tasks. An allocation to a task is considered efficient if it maximizes the worker's total expected output.

The resulting allocation problem is of course different for workers whose performance in one of the two tasks has already been observed and for new workers on whom the employer has no special information. The following two sections therefore consider these two issues in turn.

3.1 Internal Reallocation of Current Workers

Let us first consider the allocation decision of an employer who has observed workers for one period. That is, we look at the (re-)allocation problem the employer faces after the first period in Figure 1.

[8]Note that the main effects of the model may still occur if we assume a non-zero correlation between the two skills.

In order to decide whether a worker should be reallocated or not, the employer will have to calculate the worker's expected performance in either task given his performance so far.

For instance, a worker whose observed output in task 1 has been $\hat{y}_1$ can be expected to produce $E[y_2|\hat{y}_1]$ as follows if reassigned to task 2:

$$E(y_2|\hat{y}_1) = E[y_2] + k_1 \cdot (\hat{y}_1 - E[y_1]) \ , \tag{1}$$

where k_1 is defined as

$$k_1 := \frac{\alpha\beta\sigma_x^2 + (1-\alpha)(1-\beta)\sigma_z^2}{\alpha^2\sigma_x^2 + (1-\alpha)^2\sigma_z^2} = \frac{Cov(y_1, y_2)}{Var(y_1)} \ .$$

That is, k_1 is equal to the ratio between the covariance of output in tasks 1 and 2 and the variance of output in task 1. For more information on the calculation of the conditional expected output, see section 8.1 in the Appendix.

Suppose that the employer wants to maximize the overall sum of outputs y_1 and y_2. Then it is efficient to reallocate an employee if

$$E[y_2|\hat{y}_1] > \hat{y}_1 \ , \tag{2}$$

i.e., if the expected performance of a worker in task 2 is higher than his observed current performance in task 1, $\hat{y}_1$. Taking into account expression (1), one can rewrite the above inequality as

$$(k_1 - 1)\hat{y}_1 \geq k_1 E[y_1] - E[y_2] \ .$$

As a consequence, there exists a critical value $\tilde{y}_1$ of the form

$$\tilde{y}_1 = [k_1 E[y_1] - E[y_2]] \frac{1}{k_1 - 1} \ , \tag{3}$$

such that, if $k_1 > 1$, then reallocation is efficient if the observed output $\hat{y}_1$ is *higher* than $\tilde{y}_1$. Note that $k_1 > 1$ if

$$\sigma_z^2 > \frac{\alpha}{1-\alpha}\sigma_x^2 \ , \tag{4}$$

that is, if the variance of skill z in the population is high enough or α is sufficiently low or both. If this is true, i.e., if $k_1 > 1$, then the best performing workers in task 1 are likely to be even better in task 2.

Otherwise, for $k_1 < 1$, reallocation is efficient if $\hat{y}_1$ is *smaller* than $\tilde{y}_1$. Then only the worst performers in task 1 are likely to do better in task 2. This is the case if $\sigma_z^2 < \frac{\alpha}{1-\alpha}\sigma_x^2$.

By a similar reasoning, a reallocation from task 2 to task 1 is efficient if the worker's task 2 output is higher (lower) than a critical output level

$$\tilde{y}_2 = [k_2 E[y_2] - E[y_1]] \frac{1}{k_2 - 1} \,, \tag{5}$$

for k_2 larger (smaller) than 1, where k_2 is defined as

$$k_2 := \frac{Cov(y_1, y_2)}{Var(y_2)} = \frac{\alpha\beta\sigma_x^2 + (1-\alpha)(1-\beta)\sigma_z^2}{\beta^2\sigma_x^2 + (1-\beta)^2\sigma_z^2} \,.$$

Note that k_2 is larger than 1 if

$$\sigma_z^2 < \frac{\beta}{1-\beta}\sigma_x^2 \,. \tag{6}$$

By definition, one has

$$k_1 \cdot k_2 = \frac{(Cov(y_1, y_2))^2}{Var(y_1) \cdot Var(y_2)} = \varrho_{y_1 y_2}^2 \,.$$

Since the correlation coefficient $\varrho_{y_1 y_2}$ can be at most equal to one, it follows directly that the variables k_1 and k_2 cannot both at the same time be larger than 1.

Therefore the following proposition holds:

Proposition 1 *For $k_1 > 1$, there exists a critical task 1 output level $\tilde{y}_1$ as defined in (3) such that for $y_1 > \tilde{y}_1$ it is efficient to reallocate a worker from task 1 to task 2. At the same time, there exists a critical task 2 output level $\tilde{y}_2$ as defined in (5) such that for $y_2 < \tilde{y}_2$ it is efficient to reallocate workers from task 2 to task 1.*

As a corollary to this proposition, we get that if $k_2 > 1$, then it is efficient to reallocate workers with an observed output $\hat{y}_1 < \tilde{y}_1$ to task 2 and workers with $\hat{y}_2 > \tilde{y}_2$ to task 1.

Furthermore the case may arise that $k_1 < 1$ and $k_2 < 1$, and hence only the worst performers get reallocated, i.e., workers with $\hat{y}_1 < \tilde{y}_1$ and with $\hat{y}_2 < \tilde{y}_2$. This result also holds when the covariance between tasks 1 and 2 is negative.

Which of these constellations is relevant depends on the relative size of the variances of skill x and z. As we have seen, $k_1 > 1$ is equivalent to $\sigma_z^2/\sigma_x^2 > \alpha/(1-\alpha)$ and $k_2 > 1$ if $\sigma_z^2/\sigma_x^2 < \beta/(1-\beta)$. Furthermore, given Assumption 1, we know that $\alpha/(1-\alpha) > \beta/(1-\beta)$. We can thus derive three cases, which are defined as follows:

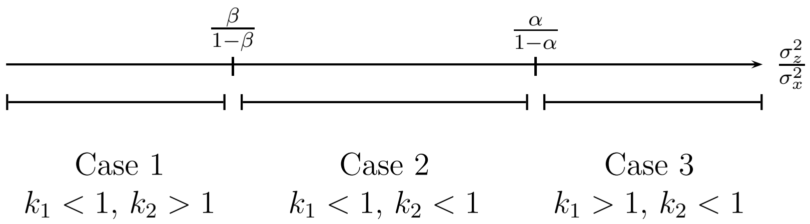

Figure 2: Possible combinations for k_1 and k_2, depending on σ_z^2/σ_x^2.

- *Case 1*: If $\sigma_z^2/\sigma_x^2 < \beta/(1-\beta)$, then $k_1 < 1$ and $k_2 > 1$.
- *Case 2*: If $\beta/(1-\beta) \leq \sigma_z^2/\sigma_x^2 \leq \alpha/(1-\alpha)$, then $k_1 < 1$ and $k_2 < 1$.
- *Case 3*: If $\alpha/(1-\alpha) < \sigma_z^2/\sigma_x^2$, then $k_1 > 1$ and $k_2 < 1$.

These cases are also summarized in Figure 2. In Cases 1 and 2, reallocation from task 1 to task 2 takes place if workers produce an output $\hat{y}_1 < \tilde{y}_1$, whereas in Case 3 workers get reallocated to task 2 if they produce $\hat{y}_1 > \tilde{y}_1$. Reallocation from task 2 to task 1 is optimal in Cases 2 and 3 if $\hat{y}_2 < \tilde{y}_2$, and in Case 1 if $\hat{y}_2 > \tilde{y}_2$.

3.2 Assignment of New Workers

So far, the reallocation of current workers has been considered. But how about new workers? To which task should they be allocated first? There are two criteria that may possibly play a role: (i) Which task provides the employer with more precise information about the employee? (ii) Where can an unscreened worker be expected to produce more?

If new workers get hired for only one period, then learning about the employee obviously does not play a role, and the employer will therefore prefer to allocate new workers to the task where the expected output of an unknown worker is higher, and thus the second motive dominates. The same is true if workers get hired for several periods, but cannot be reallocated or fired.

If workers instead get hired for two periods and reallocation is possible, then there may be a tradeoff between the two motives, i.e., between learning and output maximization of unscreened workers. To see this, let us make the following assumption, which is without loss of generality:

Assumption 2 $E[y_1] > E[y_2]$.

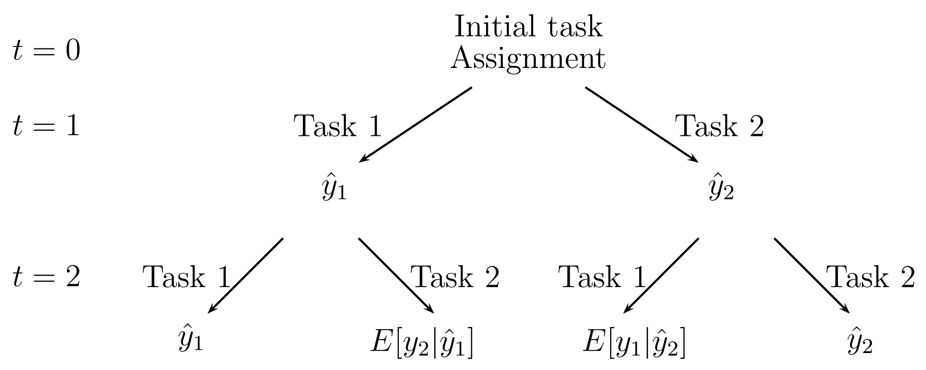

Figure 3: Decision Tree.

That is, an unscreened worker is expected to be more productive in task 1. The employer can hire the worker for either of the two tasks $j = 1, 2$. After one period of work, he will observe the worker's first period output $\hat{y}_j$. According to the rules derived in the previous section, he will then either let the worker continue to work on his current task or he will reassign him. Assuming that $k_1 > 1$ and $k_2 < 1$, this means that workers who were first assigned to task 1 get reallocated to task 2 if their conditional expected output in task 2, $E[y_2|\hat{y}_1]$, is greater than their current output, $\hat{y}_1$. As shown in Proposition 1, this is the case if $\hat{y}_1$ greater than $\tilde{y}_1$ as defined in (3). Similarly, workers who were first hired for task 2 get reassigned to task 1 if they are likely to produce a higher output in this position, i.e., if $E[y_1|\hat{y}_2] > \hat{y}_2$. This is the case if $\hat{y}_2 \leq \tilde{y}_2$ which was defined in (5). All other workers continue to work on the same task. Figure 3 illustrates these considerations.

After having observed a worker's first period output in task 1, the employer will either keep the worker in this task such that he continues to produce output $\hat{y}_1$. Or he will be reassigned if his expected output in task 2 is likely to exceed his current task 1 output, i.e., if $E[y_2|\hat{y}_1] - \hat{y}_1 > 0$. That is, the worker's expected second period output can be written as[9]

$$\hat{y}_1 + \max\left\{E[y_2|\hat{y}_1] - \hat{y}_1, 0\right\} .$$

From an ex ante point of view, the expected output of a worker hired for two

[9]Since there are no slot constraints, the worker will be assigned to the task where his output is expected to be highest, i.e., his second period output is $\max\{E[y_2|\hat{y}_1], \hat{y}_1\} = \hat{y}_1 + \max\{E[y_2|\hat{y}_1] - \hat{y}_1, 0\}$.

periods and first assigned to task 1 therefore is

$$\begin{aligned} & E[y_1] + E\Big[\hat{y}_1 + \max\{E[y_2|\hat{y}_1] - \hat{y}_1, 0\}\Big] \\ = & E[y_1] + E[\hat{y}_1] + E\Big[\max\{E[y_2|\hat{y}_1] - \hat{y}_1, 0\}\Big] \ . \end{aligned}$$

Note that $E[\hat{y}_1] = E[y_1]$, and that $E[y_2|\hat{y}_1] = E[y_2] + k_1(\hat{y}_1 - E[y_1])$, as given by equation (1). Furthermore, let $F(\cdot)$ be the cumulative distribution function of y_1.[10] Since $E[y_2|\hat{y}_1] - \hat{y}_1$ is only positive for $\hat{y}_1 > \tilde{y}_1$, as defined in (3), the above can be rewritten as

$$E[y_1] \quad + \quad E[y_1] + \int_{\tilde{y}_1}^{\infty} \Big(E[y_2] + (k_1 - 1)\hat{y}_1 - k_1 E[y_1]\Big) d\hat{y}_1$$

which is equal to

$$2E[y_1] + \Big[E[y_2] - E[y_1] + (k_1 - 1)\sigma_{y_1} \frac{\phi(a)}{1 - \Phi(a)}\Big](1 - F(\tilde{y}_1)) \ , \tag{7}$$

where σ_{y_1} is the standard error of the distribution of y_1, $F(\cdot)$ is the cumulative distribution function of y_1, $\phi(\cdot)$ and $\Phi(\cdot)$ are the probability density and cumulative density functions, respectively, of the standard normal distribution, and $a := (\tilde{y}_1 - E[y_1])/\sigma_{y_1}$. For the detailed calculations, see Section 8.2 in the Appendix.

Note that $2E[y_1]$ corresponds to the expected output of a worker assigned to task 1 if there is no learning by the employer. The second term in (7) represents the difference in expected output which is due to learning by the employer and subsequent reassignment of workers.

A similar reasoning applies when workers first work on task 2. After observing their first-period performance $\hat{y}_2$, the employer will reassign them to task 1 if $E[y_1|\hat{y}_2] > \hat{y}_2$, which is the case if $\hat{y}_2 \leq \tilde{y}_2$, and otherwise let them continue to work on task 2. From an ex ante point of view, the expected output of a worker hired for two periods and first assigned to task 2 hence is

$$\begin{aligned} E[y_2] \quad & + \quad E\Big[\hat{y}_2 + \max\Big\{E[y_1|\hat{y}_2] - \hat{y}_2, 0\Big\}\Big] \\ & = \quad E[y_2] + E[\hat{y}_2] + \int_{-\infty}^{\tilde{y}_2} \Big(E[y_1] + (k_2 - 1)\hat{y}_2 - k_2 E[y_2]\Big) d\hat{y}_2 \ . \end{aligned}$$

[10] Given the assumptions about x and z, y_1 is normally distributed with mean $\alpha\mu_x + (1-\alpha)\mu_z$ and variance $\alpha^2\sigma_x^2 + (1-\alpha)^2\sigma_z^2$.

Let $G(\cdot)$ be the cumulative distribution function of y_2.[11] Then, applying a similar reasoning as before, the above expression can be rewritten as

$$2E[y_2] + \Big(E[y_1] - E[y_2]\Big)G(\tilde{y}_2) + (k_2 - 1)G(\tilde{y}_2)\sigma_{y2}\frac{-\phi(b)}{\Phi(b)}\,, \tag{8}$$

where σ_{y_2} is the standard error of the distribution of y_2, $\phi(\cdot)$ and $\Phi(\cdot)$ are the probability density and cumulative density functions, respectively, of the standard normal distribution, and $b := (\tilde{y}_2 - E[y_2])/\sigma_{y2}$. As before, $2E[y_2]$ is the expected output of a worker assigned to task 2 for two periods, and the second term in (8) describes the difference in expected output which is due to learning about the worker's skills and optimal reassignment in accordance with this information.

By comparing (7) and (8), we can derive the following proposition:

Proposition 2 *Given Assumptions 1 and 2, if $k_1 > 1$, then an employer who hires workers for two periods of time will prefer to hire them first for task 1 if (7) is greater than (8). Otherwise, new workers get first assigned to task 2.*

That is, even if $E[y_1] > E[y_2]$ the employer will consider hiring workers for task 2 first, if his benefit from learning about the employee in this task is high enough to compensate for the loss of expected output.[12]

This tradeoff is also found in the experimentation literature. For instance, Grossman, Kihlstrom, and Mirman (1977) show that individuals or firms may find it profitable to modify their consumption behavior in order to base their future consumption decisions on better information. That is, they experiment in order to gain information.[13]

The same is true in the present model. It would seem natural that the employer should assign workers to the job where they can be expected to produce the highest output.

However, if the opportunity costs are not too high and if performance in task 2 is much more informative about workers' skills, then the employer may find it worthwhile to assign workers first to task 2, thus basically buying

[11] y_2 is normally distributed with mean $\beta\mu_x + (1-\beta)\mu_z$ and variance $\beta^2\sigma_x^2 + (1-\beta)^2\sigma_z^2$.

[12] A numerical example where this situation arises can be found in Appendix 8.2.

[13] Jovanovic and Nyarko (1997) also find that "safe" activities, i.e. activities where mistakes are likely to destroy less output, can be used as a training ground. Note, however, that unlike in the present model, in Jovanovic and Nyarko (1997) there is on-the-job-training of employees and no learning by the employer.

information on workers at the cost of a lower expected output in the short run.

Or, in the terms of the model by Grossman et al. (1977): the employer decides to "experiment" if the benefits from more informed future choices outweigh the costs incurred because he modifies his behavior relative to what would be optimal if there was no learning.[14]

Note, however, that, while it is possible that such "experimentation" arises, it is not necessarily the case. Depending on the exact distributions of y_1 and y_2, there is not necessarily a tradeoff between maximizing the first-period output of an unknown worker and learning about his skills. In that case, the employer will be better off assigning new workers to task 1 where they are expected to produce a higher output.

4 Job Assignment with Slot Constraints

The previous section has established criteria for assigning both new and old workers to one of two activities in the firm. These criteria are relevant if the employer faces no constraints concerning the number of employees working on each task. However, the employer may not be able to assign any number of workers to any task. At least in the short run he is likely to face slot constraints in the sense that he needs a fixed number of employees in each activity. These constraints may be more important due to labor laws or in jobs where physical capital matters more, i.e. for blue collar workers.

Let us assume that the firm needs n workers to do task 1 and m workers for task 2. That is, there are n jobs of type 1 and m jobs of type 2. All prices are normalized to one and the firm's objective function is given by

$$\sum_{i=1}^{n} y_1^i + \sum_{i=1}^{m} y_2^i \ ,$$

i.e., the principal wants to maximize the sum of outputs while keeping the number of workers in each task fixed.

In the following, only the employer's short-term perspective is considered, i.e., he wants to maximize the sum of outputs in the next period. To fill all available slots, the employer can either reallocate current workers or hire new

[14] Similar behavior may be found on the side of employees: as Antonovics and Golan (2012) show, if workers don't know their type, they may also engage in experimentation in order to learn more about their skills.

unscreened workers for either of the two jobs. So when does the firm choose which option?

4.1 External Recruiting vs. Internal Reallocation

Suppose the firm has to fill one open position in job 2.[15] Should it fill the position in job 2 with someone who has worked in job 1 before, or should it hire an unknown worker?

If the firm promotes worker i from job 1 to job 2 and fills the opening in job 1 with an unknown worker, its expected profit in the next period is

$$E[y_1] + E[y_2^i|\hat{y}_1^i] \ ,$$

i.e., the sum of the expected output of an unknown worker in job 1 and the expected output of worker i in job 2, conditional on his *observed* performance $\hat{y}_1^i$ in his current job.

If the firm does not promote worker i and hires an unknown worker for job 2, its expected profits are

$$\hat{y}_1^i + E[y_2] \ ,$$

i.e., worker i will continue to produce the same output $\hat{y}_1^i$ as before, and the new worker has expected job 2 output $E[y_2]$.

Promoting worker i is hence the better option if

$$\begin{aligned} E[y_1] + E[y_2^i|\hat{y}_1^i] &> \hat{y}_1^i + E[y_2] \\ \Leftrightarrow E[y_1] + E[y_2] + k_1(\hat{y}_1^i - E[y_1]) &> \hat{y}_1^i + E[y_2] \\ \Leftrightarrow (k_1 - 1)\hat{y}_1^i &> (k_1 - 1)E[y_1] \ . \end{aligned}$$

As before, one has to distinguish two cases: if $k_1 > 1$, then promotion is profitable for $\hat{y}_1^i > E[y_1]$. In the opposite case, i.e., if $k_1 < 1$, promotion is only profitable if the current output is below the expected output in this job, i.e., if $\hat{y}_1^i < E[y_1]$. The analogue reasoning applies for job 2.

Proposition 3 *For $k_1 > 1$, rather than hiring a new worker, the employer prefers to reallocate a current worker from job 1 to job 2 if his output $\hat{y}_1 > E[y_1]$, and from job 2 to job 1 if $\hat{y}_2 < E[y_2]$.*

As a corollary, we get that for $k_2 > 1$ reallocation from job 1 to 2 is profitable if $\hat{y}_1 < E[y_1]$ and from job 2 to 1 if $\hat{y}_2 > E[y_2]$. If both k_1 and k_2 are smaller than one, then reallocation is profitable for $\hat{y}_1 < E[y_1]$ and $\hat{y}_2 < E[y_2]$.

[15] If there is an opening in job 1, the employer's considerations run along the same lines as those outlined in the following.

4.2 Internal Recruiting and the Structure of the Firm

The results so far suggest that internal hiring becomes more likely if the number of positions n and m to fill in task 1 and 2 is very uneven. Suppose, for instance, that $k_1 > 1$ holds[16] and hence above average workers in task 1 (i.e., with output $\hat{y}_1 > E[y_1]$) are candidates for reallocation to task 2. The chances that one of the current job 1 workers is above average increases of course with the number of workers n in this job. As a consequence, it becomes more likely that all job 2 workers are recruited internally, if n is relatively high and m relatively low.

Supposing that job 2 ranks above job 1 in a hierarchy,[17] this finding is in line with the empirical literature on internal recruiting, which generally finds that larger firms are more likely to recruit internally. DeVaro and Morita (2009) also show that this is particularly true in more "bottom heavy" firms, i.e., with a lot more workers on lower levels of the hierarchy. A similar finding is also reported by Hutchens (2006), who examines why firms sometimes employ older workers, but tend to not hire new older workers for the same job. He shows that older workers often hold jobs that are simply not filled from the outside, and that this phenomenon is more likely to occur in firms with a larger number of workers in lower-level jobs.

4.3 The Peter Principle

The paper has derived two rules for the reassignment of workers: while Section 3.1 has focused on the optimal reassignment of workers without slot constraints, Section 4.1 considered reassignment when there is a limited number of positions in each task that have to be filled.

When we compare the two resulting reassignment rules, it becomes clear that they usually do not coincide: if there are no constraints on the number of workers in each task, the optimal assignment of a worker currently employed in task $j = 1, 2$ depends on his performance relative to a critical output level $\tilde{y}_j$ as summarized in Proposition 1. By contrast, if there are slot constraints, what matters is just the worker's performance relative to the average performance of an unscreened worker, i.e., $E(y_j)$, as shown in Proposition 3.

[16] This corresponds to Case 3 as defined in Section 3.1.

[17] Note that the model is a priori agnostic concerning the position of job 1 and job 2 in the hierarchy of the firm and thus applies both to vertical and to lateral movements across the hierarchy.

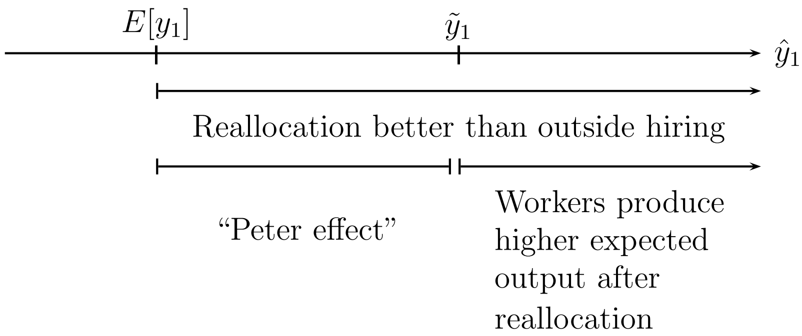

Figure 4: Individual efficiency and profitability of reallocation for $k_1 > 1$.

Take, for example, the case where $k_1 > 1$ and Assumption 2 holds, i.e., $E[y_1] > E[y_2]$.[18] In this case, which is illustrated in Figure 4, $\tilde{y}_1 > E[y_1]$. Any worker producing an output above $E[y_1]$ is a candidate for reallocation according to Proposition 3. However, only workers whose output also exceeds $\tilde{y}_1$ can be expected to be more productive after reallocation according to Proposition 1. That is, workers producing output $\tilde{y}_1 > \hat{y}_1 > E[y_1]$ are candidates for reallocation according to Proposition 3, even though they are likely to be less productive after reallocation than they are in their current job.

This immediately brings to mind the well-known Peter Principle,[19] which states that workers get promoted up to their level of incompetence. If this principle applies in the present model, then after reallocation, there must be some workers who are less competent or less productive in their new job than they were in their previous job.

This "Peter effect" can indeed be found for workers who get reallocated from job 1 to job 2 if Assumption 2 holds, no matter what the size of of k_1 and k_2.[20] Workers producing an output level between $\tilde{y}_1$ and $E[y_1]$ will be candidates for reallocation to job 2, even though they are likely to perform worse after reallocation. In other words: given their observed performance in job 1, these workers are likely to be more productive or "more competent" in their current

[18] This corresponds to Case 3 in the above.

[19] After Peter and Hull (1969). Note that, in its original version, the Peter Principle is a statement on movements across hierarchy levels, while the present model is agnostic about the rank of job 1 and 2 within a hierarchy. That is, an effect like the one suggested by the Peter Principle may be even present when workers are reallocated on the same level.

[20] Note that if Assumption 2 does not hold, then the Peter effect appears for workers who are candidates for reallocation from job 2 to job 1. The effect only depends on the relative size of $E[y_1]$ and $E[y_2]$.

job. Nevertheless, they may get transferred to job 2. The model therefore predicts indeed that there are some workers who get reallocated although they can be expected to be less productive or less competent afterwards, as suggested by the Peter Principle.

Proposition 4 *For $j = 1, 2$ and $E[y_j] > E[y_{-j}]$, workers in job j who produce an output between $\tilde{y}_j$ and $E[y_j]$ are candidates for reallocation to job $-j$ even though they are expected to be less productive after reallocation.*

This effect becomes more relevant the further apart are the two critical values $E[y_j]$ and $\tilde{y}_j$ that determine reallocation from job j to job $-j$. The distance between these two values is bigger the lower the coefficient k_j, and the larger the difference in expected output between the two jobs, i.e., the larger $|E[y_1] - E[y_2]|$.

In the previous literature on the Peter Principle, Fairburn and Malcomson (2001) discuss how promotions can limit the effect of influence activities of workers and show that depending on the degree of risk aversion, promotions may take place that are not justified by reasons of job assignment (what the authors refer to as the "Peter Principle effect"). In Lazear (2004), the Peter Principle is explained as follows: workers' performance depends both on ability and other random factors such as luck which exhibit regression to the mean. As a consequence, workers' expected performance after promotion is lower than their observed performance on which the promotion decision is based. Furthermore, after promoting the best job 1 workers, the average performance of job 1 workers left behind is lower than it was before. That is, ability appears lower after promotion purely as a statistical matter.

The present model proposes a different explanation. It shows that workers get reallocated as soon as they produce more than the average unscreened worker, even if they are likely to be less able and hence less productive after promotion. This is due to the fact that the employer faces an opportunity cost of hiring an unknown worker, and therefore always prefers to fill an open position with a worker on whom he has at least some (positive) information.

Note however, that this is an efficient assignment policy. The Peter Principle here occurs as a by-product of a labor market where workers' skills are difficult to assess. If the employer's information on outside candidates[21] improves, the scope for the Peter effect as described above diminishes.

[21] For instance, the employer may be able to observe past performance of workers with a different employer or he may get a signal on a worker's skill level in one of the relevant skills.

4.4 Robustness of Results

The Peter effect described above springs from the different quality of information on firm insiders and outsiders. The employer has slightly better, but not perfect information on the skills of insiders which may lead to the promotion of the latter even if they are expected to perform worse after promotion. If the firm has better information on outsiders, the effect diminishes.

So what happens, for instance, if the firm is one of many, all facing the same problem? There are reasons to believe that while competition between firms may have some effect on the information available on outsiders, in most cases it will not influence the essence of the results above. First of all, skills may be firm- or task-specific, as suggested by Gibbons and Waldman (2004) and Lazear (2009). If this is the case, an outside worker is less attractive, even if the labor market should become more transparent. The scope for the Peter Principle will rather increase if such effects are taken into account since the promotion decision is even more biased towards internal candidates. Second, firms may have strategic reasons to limit the information that is available to their competitors, thus making the labor market less transparent.[22] This aspect has for instance been analyzed in Waldman (1984) who shows that such strategic behavior is a further source of biased job assignment that is even more important if firm-specific skills play a smaller role and competition for potential job candidates thus is stronger.

Yet not only the information on outsiders matters for the Peter effect, but also the information on insiders. The Peter Principle describes a mismatch between the skills of a worker and the job requirements, which is revealed after a promotion. Thus, the better the information on insiders' skills, the lower should be the scope for the Peter effect. To analyze this further, let us extend the model considered above to three periods and illustrate our results using the same example as before, namely a scenario where $E[y_2] < E[y_1]$ and $k_1 > 1$, and therefore $E[y_1] < \tilde{y}_1$. As we have seen in the previous section, a worker who produced a first period output $\hat{y}_1 > E[y_1]$ is a candidate for promotion if there is an opening in job 2, but only a worker with $\hat{y}_1 > \tilde{y}_1$ can be expected to be more productive after promotion. That is, there is a Peter effect for all candidates with $E[y_1] < \hat{y}_1 < \tilde{y}_1$: they are candidates for promotion even though their output is expected to be lower afterwards.

Now what happens after the second period, if the employer has observed the output of a promoted worker in both tasks? If the worker's output in job 2

[22]Friebel and Raith (2004) show that even within the firm the information exchange on employees may be limited for strategic reasons.

is higher than in job 1, then the promotion was justified, the assignment is efficient and no Peter effect occurs. But what happens if $\hat{y}_2 < \hat{y}_1$, i.e., if the assignment turns out to be inefficient?

In the basic model, the employer now knows the worker's skill profile perfectly after having seen his performance in both tasks. In the absence of slot constraints, the worker therefore could be assigned to the task where he is most productive.[23] That is, as before, there is no room for a Peter Principle effect if there are no slot constraints. However, if there are slot constraints or if demotion is difficult for other reasons, then the firm will want to keep the worker in job 2 as long as his output in this job is at least higher than that of an unscreened candidate, i.e., if $\hat{y}_2 > E[y_2]$.[24] The employer thus prefers to keep a worker in job 2 even though he *knows* that his output would be higher in job 1. Even with complete knowledge of the worker's skill profile, slot constraints may therefore lead to inefficient job assignments that reflect the Peter Principle in the sense that output after promotion may be lower than before.

Note that if the firm could always perfectly observe the worker's skills, it could assign the worker to the task where he is most productive in the first place. However, this assumption is not very realistic since the output of a worker in general will depend on many factors besides his own skills, luck being one of them. In the following section, I therefore generalize the model to include a stochastic component. Furthermore, output in one job may also depend on output produced by others. This aspect is discussed in Section 5.2.

5 Generalization of the Model

5.1 Production Function of the Worker

In the basic model described above the weights of skills in a given task always summed up to one, thus putting the focus on the relative importance of skills in each task. However, all the results still hold if this is not true. Also, the model can be further generalized to encompass both more tasks, skills, and periods, as well as external shocks: suppose that there are $j = 1, .., J$ possible tasks and $s = 1, .., S$ skills. Each skill has a different weight c_{sj} in each task.

[23]Note that a promoted worker is above average at least in one job, here in job 1, which means that the firm has an interest to keep him in the firm.

[24]If we allow for firing, workers with $\hat{y}_2 \leq E[y_2]$ will be fired, unless they can be demoted to job 1 where they are more productive than an unscreened worker.

Furthermore let y_j denote the output in task j. The production function for task j then can be written as

$$y_j = \sum_{s=1}^{S} c_{sj} \cdot x_s + \epsilon_j \ ,$$

where x_s denotes the endowment of a worker with skill s, and ϵ is a noise term. Skills are assumed to be independently distributed in the population according to $N(\mu_{x_s}, \sigma_{x_s})$. The noise term is also assumed to be independently distributed across tasks and across time periods according to $N(0, \sigma_\epsilon)$. This generalization allows for external productivity shocks, which makes it better comparable to existing models of job assignment. Due to the error term, the employer never learns the exact skill profile of a worker even after several periods, but can only update his beliefs.

The analysis from the previous section also applies to this more general version of the model: given an observed output in task j, $\hat{y}_j$, the expected output of a worker in task $k \neq j$ is

$$E[y_k|\hat{y}_j] = E[y_k] + \frac{Cov(y_j, y_k)}{Var(y_j)} \cdot [\hat{y}_j - E[y_j]] \ . \tag{9}$$

If kept in the same task for another period, the worker has an expected output of

$$E[y_j|\hat{y}_j] = E[y_j] + \frac{Cov(y_j, y_j)}{Var(y_j)} \cdot [\hat{y}_j - E[y_j]] \ , \tag{10}$$

where – in a slight abuse of notation – $Cov(y_j, y_j)$ denotes the covariance of output in the same task across periods.

If there are no slot constraints, then a worker should be reallocated to task k whenever his expected output in task k as given in (9) is greater than his expected output in task j, as given in (10). That is, as before, one can determine an output level $\tilde{y}_{(j,k)}$ that is critical for the decision whether a worker should be kept in task j or whether he can be expected to be more productive in task k.

If, however, there are slot constraints and the employer has to decide between hiring new workers and worker reallocation, then this decision is determined by workers' performance in task j, $\hat{y}_j$, relative to the expected performance of a new worker, $E[y_j]$.

That is, as in the basic model, we get two critical values that drive the reallocation decision, depending on whether there are slot constraints or not, namely $\tilde{y}_{(j,k)}$ and $E[y_j]$. These two values will usually not coincide, and we thus can get the same effects as in the basic model. In particular, the Peter effect described above may still occur.

A further generalization of the model would be to look not only at general skills, but also at firm-specific or even job-specific skills that are acquired through experience. To keep the model simple and underline the main effect, such learning by the employee has been excluded. However, it should be clear that if firm-specific human capital plays a role this will further reduce the incentive to hire complete outsiders, making it more likely that insiders are promoted, whereas if there is on the job learning, this will increase the output of a worker in the same job over time, thus making it relatively more likely that he will keep this job.

5.2 Objective Function of the Firm

So far, we have assumed that the firm merely wants to maximize the sum of outputs, thus abstracting entirely from costs. A straightforward way to account for this would be to rather consider the weighted sum of outputs, where the weights represent profit shares. That is, in the case with just two jobs, the objective function of the firm would now be:

$$\delta \sum_{i=1}^{n} y_1^i + (1-\delta) \sum_{i=1}^{m} y_2^i \ ,$$

where δ reflects the profit share that can be attributed to job 1.[25] While this has an effect on the exact critical values of output[26] that influence the reallocation decision, the results derived before still hold. In particular, the critical output levels for reallocation with and without slot constraints still may differ, thus leaving room for a Peter Principle effect to occur.

When we formulate the firm's objective function, we may also need to take into account the structure of an organization and the interdependence of

[25] Since the main costs are likely to be wage costs, alternatively, one could formulate the objective function as $\sum_{i=1}^{n}(y_1^i - w_1) + \sum_{i=1}^{m}(y_2^i - w_2)$. With given job-specific market wages, the interpretation then is quite similar to the formulation as profit shares, since the main determinant of total profits will still be an optimal assignment to jobs. The essence of the results thus still applies. Things may change if we allow for performance-based wages. However, this goes beyond the scope of the present paper, which abstracts explicitly from incentive issues to concentrate on the pure assignment aspect of the problem.

[26] In particular, the critical output level without slot constraints now would be $\tilde{y}_1 = (k_1 E(y_1) - E(y_2))(1-\delta)/((1-\delta)k_1 - \delta)$.

outputs. For instance, the output in job 1 may depend on output in job 2. This could be interpreted as a production chain. Or else one could imagine that job 2 is a manager, and overall production in the unit depends on him giving the right orders.[27] The overall production then may not be best described by the sum of outputs in each task, but rather by a weighted sum of outputs, as discussed above, or by including some form of interaction term. The resulting complementarity of outputs across hierarchy levels is for instance also used in Williamson (1967), Calvo and Wellisz (1978, 1979), Qian (1994) and Friebel and Raith (2004).

Again this modification of the firm's objective function will result in different critical values of output, yet it does not change the main results of the paper. As before, the firm will be better informed about workers it has observed for one period which, for a certain output range, may make it preferable to promote an internal candidate with intermediate performance rather than hiring an external candidate on whom the firm has no information at all. That is, depending on the exact parameter values, the Peter Principle may still hold.

6 Career Paths

In the model presented here, the employer can only learn more about a worker's type if he assigns the worker to different jobs. This raises the questions how an employer wants to structure a possible career path. Will consecutive jobs be closely related to each other? When is job rotation a useful mechanism?

Consider the following specification of the model: there are four jobs that use four different skills as follows:

$$\begin{aligned} y_1 &= c_{11}x_1 + c_{12}x_2 \\ y_2 &= c_{22}x_2 + c_{23}x_3 \\ y_3 &= c_{33}x_3 + c_{34}x_4 \\ y_4 &= c_{41}x_1 + c_{42}x_2 + c_{43}x_3 + c_{44}x_4 \end{aligned}$$

Suppose that job 4 is a management job. In order to choose candidates for this job, the employer faces the option of letting a worker move gradually

[27] While the Peter Principle is usually discussed in the context of promotions, the mechanisms under consideration in this paper are more general. They apply both to vertical and lateral movements in a hierarchy. As has been shown by Baker, Gibbs, and Holmström (1994), a career in a large organization is likely to encompass both types of movements. See also the following section for a discussion.

through jobs 1, 2, and 3, i.e., structure his career path as a *job ladder.* Or he may want to let workers do job 1 and 3 before moving them to job 4, i.e., implement a *job rotation* between quite different jobs (1 and 3).

The latter structure allows the employer to extract the most information about the worker in just two time periods. However, switching a job 1 worker to job 3 promises the same expected job 3 output as hiring a completely new worker for the job. Also, by letting a worker work his way through job 1 to 3, the employer gets more observations and hence a better estimate of the worker's skills.

A job ladder hence provides the employer with a more thorough candidate screening, but it also takes more time. A job rotation program, by contrast, is a speedier way to collect some information about aspiring managers, though at the cost of possibly getting a very low performance of workers in job 3.

This cost can, however, be mitigated if the employer has better information about workers before hiring them for a job rotation program. Supposing workers can signal their skills through education, previous work, extracurricular activities, and so on, workers with good signals from various fields therefore seem to be more likely to be selected into job rotation schemes, such as, for example, high-profile trainee programs.[28]

Of course, the analysis in this paper also neglects a second aspect that may play an important role here, namely that the structuring of a career path also affects the learning process of the employee. This aspect is analyzed in Jovanovic and Nyarko (1997) as well as in Gibbons and Waldman (2004), who introduce task-specific human capital that is acquired through learning by doing.[29] Reallocating workers to similar jobs, i.e., specialization, hence makes the most efficient use of this capital. However, for a manager it may be more important to be somewhat knowledgeable in several fields, rather than being an expert in just one, which would explain the existence of job rotation.

So which of these two explanations – learning by the employer or learning by the employee – goes the longer way in explaining why firms adopt job rotation practices? The papers by Ortega (2001) and Eriksson and Ortega (2006)

[28]This would correspond to the results of the studies by Campion, Cheraskin, and Stevens (1994) and Kusunoki and Numagami (1998), which both suggest that rotation may be good for a worker's career and possibly is used to generate the promotion pool for new managers.

[29]Note that this is an important difference to the present model, where workers' skill levels are given from the outset and at best may be improved through learning by doing.

consider exactly this question.[30] While they find evidence that both employer and employee learning may play a role, the evidence for the former seems slightly stronger. In particular, the authors find that tenure in the firm has a significant negative effect on rotation, whereas tenure in the industry does not, thus suggesting that rotation is a means to get to know the employee, rather than a training device. Furthermore, firms with more hierarchy levels and broader recruiting strategies, as well as growing firms, are more likely to adopt job rotation, all of which supports the employer learning hypothesis.

7 Conclusion

The paper proposed a simple task assignment model with multidimensional skills and derived conditions, under which the employer can increase output by reallocating workers. However, there is a potential tradeoff between short-term output maximization and learning about the employee's skills: The employer may want to hire new workers to a task with lower expected output if the expected loss of output thus generated is not too high while at the same time he gets better information on the employee's skills.

This first part of the paper thus looked at the optimal assignment of a given worker if the employer is free to assign him to any task he sees fit. By contrast, the second part considered a setting where the employer has to fill a given job with a worker which he may choose from within or outside the firm. In such a setting, workers may get reassigned to another job even if, in expectation, they will be less productive after reassignment. This simply arises because employers prefer to reallocate workers on whom they have some information, rather than hire completely new workers on the market.

The paper thus provides a new explanation for the Peter Principle: workers may indeed get reallocated in such a way that, in the end, they are actually less suited for their current than for their previous job. However, this policy is efficient, since the employer otherwise has to fill his open positions with new applicants on whom he has less information than on his current workers. The Peter Principle thus arises as a by-product of insufficient knowledge about outside candidates and its relevance varies with the availability of information on workers.

[30] Ortega (2001) and Eriksson and Ortega (2006) also identify a third motive for job rotation, namely motivating employees by mitigating boredom. However, they find little evidence in support of this motive.

Whether this effect plays a role therefore depends crucially on the transparency of the relevant labor market. The transparency of the labor market, in turn, and hence the information available on candidates, depends on market structures, the degree of competition, and the availability of reliable signals on workers' capabilities. While these aspects are beyond the current model, it may be interesting to explore them further.

8 Appendix

8.1 Conditional Expectations

Conditional Normal Distributions: General Theorem

How to calculate the conditional expectation of x and z? A nice summary is provided in Greene (2003): Let $\mathbf{x} = (x_1, x_2, \cdots, x_n)'$ be a vector of n random variables with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. Let $\mathbf{x_1}$ be any subset of variables, and let $\mathbf{x_2}$ be the remaining variables. Likewise, partition $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ so that

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$$

and

$$\boldsymbol{\Sigma} = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} .$$

Then the conditional distribution of $\mathbf{x_1}$ given $\mathbf{x_2}$ is normal as well:

$$\boldsymbol{x_1} | \boldsymbol{x_2} \sim N(\mu_{1.2}, \Sigma_{11.2}) ,$$

where

$$\begin{aligned} \mu_{1.2} &= \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(x_2 - \mu_2) , \\ \Sigma_{11.2} &= \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} \end{aligned}$$

Application to the Production Functions

Consider the production functions $y_1 = \alpha x + (1-\alpha)z$ and $y_1 = \alpha x + (1-\alpha)z$. Variables x and z are independently normally distributed with $x \sim N(\mu_x, \sigma_x)$ and $z \sim N(\mu_z, \sigma_z)$. Hence the mean vector of $(y_1, y_2)'$ corresponds to

$$\boldsymbol{\mu} = \begin{pmatrix} \alpha\mu_x + (1-\alpha)\mu_z \\ \beta\mu_x + (1-\beta)\mu_z \end{pmatrix}$$

and the variance matrix is given by

$$\boldsymbol{\Sigma} = \begin{pmatrix} \alpha^2\sigma_x^2 + (1-\alpha)^2\sigma_z^2 & \alpha\beta\sigma_x^2 + (1-\alpha)(1-\beta)\sigma_z^2 \\ \alpha\beta\sigma_x^2 + (1-\alpha)(1-\beta)\sigma_z^2 & \beta^2\sigma_x^2 + (1-\beta)^2\sigma_z^2 \end{pmatrix} .$$

Following the above-mentioned theorem, the conditional distribution of y_1 given y_2 can be characterized as $y_1|y_2 \sim N(\mu_{1.2}, \Sigma_{11.2})$. That is, the expectation of y_1 given the observed value of y_2, i.e. $\mu_{1.2}$ or $E[y_1|\hat{y}_2]$, is

$$
\begin{aligned}
\mu_{1.2} &= \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(y - \mu_2)) \\
&= [\alpha\mu_x + (1-\alpha)\mu_z] + \frac{\alpha\beta\sigma_x^2 + (1-\alpha)(1-\beta)\sigma_z^2}{\beta^2\sigma_x^2 + (1-\beta)^2\sigma_z^2}\Big(\hat{y}_2 - [\beta\mu_x + (1-\beta)\mu_z]\Big) \\
&= E[y_1] + \frac{Cov(y_1, y_2)}{Var(y_2)}(\hat{y}_2 - E[y_2]) \; .
\end{aligned}
$$

8.2 Calculating the Ex Ante Expected Output of New Workers

From an ex ante point of view, the expected output of a worker hired for two periods and first assigned to task 1 is

$$
\begin{aligned}
& E[y_1] + E\Big[\hat{y}_1 + \max\{E[y_2|\hat{y}_1] - \hat{y}_1, 0\}\Big] \\
&= E[y_1] + E[\hat{y}_1] + E\Big[\max\{E[y_2|\hat{y}_1] - \hat{y}_1, 0\}\Big] \; .
\end{aligned}
$$

Note that $E[\hat{y}_1] = E[y_1]$, and that $E[y_2|\hat{y}_1] = E[y_2] + k_1(\hat{y}_1 - E[y_1])$, as given by Equation (1). Furthermore, let $F(\cdot)$ be the cumulative distribution function of y_1. Since $E[y_2|\hat{y}_1] - \hat{y}_1$ is only positive for $\hat{y}_1 > \tilde{y}_1$, as defined in (3), the above can be rewritten as

$$
\begin{aligned}
E[y_1] &+ E[y_1] + \int_{\tilde{y}_1}^{\infty}\Big(E[y_2] + (k_1 - 1)\hat{y}_1 - k_1 E[y_1]\Big)d\hat{y}_1 \\
&= 2E[y_1] + \Big(E[y_2] - k_1 E[y_1]\Big)(1 - F(\tilde{y}_1)) + (k_1 - 1)\int_{\tilde{y}_1}^{\infty}\hat{y}_1 d\hat{y}_1 \\
&= 2E[y_1] + \Big(E[y_2] - k_1 E[y_1]\Big)(1 - F(\tilde{y}_1)) \\
&\qquad + (k_1 - 1)(1 - F(\tilde{y}_1))E[y_1|y_1 > \tilde{y}_1] \; .
\end{aligned}
$$

To calculate the conditional expectation in the last line, we need the following result on truncated normal distributions: Let x be a normally distributed random variable with $x \sim N[\mu, \sigma]$ and let c be a constant, then

$$
\begin{aligned}
E[x|x > c] &= \mu + \sigma\frac{\phi(\gamma)}{1 - \Phi(\gamma)} \\
&\text{and} \\
E[x|x < c] &= \mu + \sigma\frac{-\phi(\gamma)}{\Phi(\gamma)} \; ,
\end{aligned}
$$

where $\gamma := (c - \mu)/\sigma$ and $\phi(\cdot)$ is the standard normal density.

Therefore,

$$E[y_1|y_1 > \tilde{y}_1] = E[y_1] + \sigma_{y_1} \frac{\phi(a)}{1 - \Phi(a)} ,$$

where σ_{y_1} is the standard error of the distribution of y_1, $\phi(\cdot)$ and $\Phi(\cdot)$ are the probability density and cumulative density functions, respectively, of the standard normal distribution, and $a := (\tilde{y}_1 - E[y_1])/\sigma_{y_1}$.

Taking all this into account, the expected output of a worker hired for two periods and starting in task 1 can be rewritten as

$$\begin{aligned} 2E[y_1] \quad &+ \quad \Big(E[y_2] - k_1 E[y_1]\Big)(1 - F(\tilde{y}_1)) \\ &+ \quad (k_1 - 1)(1 - F(\tilde{y}_1))\Big(E[y_1] + \sigma_{y_1} \frac{\phi(a)}{1 - \Phi(a)}\Big) , \end{aligned}$$

which is equal to

$$2E[y_1] + \Big[E[y_2] - E[y_1] + (k_1 - 1)\sigma_{y_1} \frac{\phi(a)}{1 - \Phi(a)}\Big](1 - F(\tilde{y}_1)) .$$

An analogue reasoning applies for the calculation of ex ante expected output when workers are first assigned to task 2.

Numeric Example

At first sight, it may not be obvious that the case can arise where the employer wants to assign workers first to task 2, even though expected output in this task is lower. To see that this is indeed possible, consider the following numeric example: Suppose that $x \sim N(1, 1)$, $z \sim N(0.8, 2)$, $\alpha = 0.7$ and $\beta = 0.5$. Then the unconditional expected output in task 1 is $E[y_1] = 0.94$ and in task 2 it is $E[y_2] = 0.9$. However, the ex ante expected payoff of hiring a worker first to task 2 and then reassigning him in an optimal way is 1.95, whereas it is only 1.92 if the worker is first hired to task 1.

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